Conference GEOEDP 2025
University of Pisa, 7-11 April 2025

Titles and abstracts

Lectures by **Franco Flandoli (SNS Pisa)** on Transport, waves and diffusion in the Charney-Hasegawa-Mima model (3x1h30)

Lecture 1. Physical interpretations of (C)HME. Invariants, waves and other special solutions of HME. Point vortex approximation. White noise solution (distributional random initial condition).

Lecture 2. Speculation about turbulence, its interaction with waves and zonal flows, in the HME ; transport and diffusion of temperature by the HM flow. Stochastic modeling of the flow. A theorem on quantitative diffusion approximation..

Lecture 3. Proof of the quantitative diffusion approximation. Other mixing properties, differences for fractional models ; streamers and transport barriers.

Talks :

Federico Cacciafesta and Elena Danesi (Univ. Padova and Politecnico di Torino)

Dispersive estimates for the Dirac equation with critical potentials : Part 1 and Part 2

Dispersive equations perturbed with electric and magnetic potentials have attracted considerable attention in the years. In this two-parts talk, we will present recent results concerning the validity of dispersive estimates, in particular generalized Strichartz estimates, for the massless Dirac equation in space dimension 2 and 3, perturbed with critical potentials. In this context, the criticality is given by the -1 homogeneity of the potentials which let the whole equation scaling invariant. This kind of invariance prevents, typically, the use of perturbative methods in the study of the effect of these potentials, requiring the development of new tools, like a "relativistic" version of the Hankel transform. Particularly meaning examples, from the point of view of physical applications, are given by the electric Coulomb (Part 1) and the magnetic Aharonov-Bohm potentials (Part 2).

Nicolas Camps (Univ. Rennes)

Probabilistic well-posedeness for the nonlinear Schrödinger equation on the 2-sphere

This talk is devoted to a probabilistic approach to the cubic nonlinear Schrödinger equation (NLS) on the two-dimensional sphere, where we study the collective behavior of random initial data supported below the deterministic threshold for the Cauchy theory.

Andreia Chapouto (Univ. Versailles)

Pathwise well-posedness of stochastic nonlinear dispersive equations with multiplicative noises

Over the last decades, the well-posedness issue of stochastic dispersive PDEs with multiplicative noises has been extensively studied. However, this study was done primarily from the viewpoint of Ito solution theory, and pathwise well-posedness remained completely open. In this talk, I will present the first pathwise well-posedness results for stochastic nonlinear wave equations and stochastic nonlinear Schrödinger equations with multiplicative white-in-time/coloured-in-space noise. Here, we combine the operator-value controlled rough paths adapted to dispersive flows, together with random tensor estimates, and the Fourier restriction norm method.

Alix Deleporte (Univ. Paris-Saclay)

Spectral and heat observability on hyperbolic surfaces

Eigenfunctions of the Laplacian cannot vanish on a set of positive measure. Quantitative versions of this unique continuation are well-known on fixed Riemannian manifolds : the L^2 norm of an eigenfunction is bounded by its L^2 norm on a set of positive measure times a constant which grows exponentially with the frequency. This growing rate is sharp and reflects in observability properties for the heat equation.'In this talk, I will present recent results, in collaboration with M. Rouveyrol (Orsay) about non-compact hyperbolic surfaces. Quantitative unique continuation, and observability of the heat equation, hold under a necessary and sufficient condition of thickness of the observed set : it must intersect every large enough metric ball with a mass bounded from below, proportionally to the mass of the ball itself. The proof crucially uses the Logunov-Mallinikova estimates.

Louise Gassot (Univ. Rennes)

Zero-dispersion limit for the Benjamin-Ono Equation

We focus on the Benjamin-Ono equation on the line with a small dispersion parameter. The goal of this talk is to precisely describe the solution at all times when the dispersion parameter is small enough. This solution may exhibit locally rapid oscillations, which are a manifestation of a dispersive shock. The description involves the multivalued solution of the underlying inviscid Burgers equation, obtained by using the method of characteristics. This work is in collaboration with Elliot Blackstone, Patrick Gérard and Peter D Miller.

Felice Iandoli (Univ. Calabria)

Strong ill-posedness in L^∞ of the 2D Boussinesq equations

In this talk I will present a work in which the strong ill-posedness of the two-dimensional Boussinesq system is proven. I will show explicit examples of initial data with vorticity and density gradient in $L^\infty(\mathbb{R}^2)$ for which the horizontal density gradient exhibits a strong norm inflation in infinitesimal time. This is a joint work with Roberta Bianchini (CNR) and Lars Eric Hientzsch (Karlsruhe institute of Technology).

Mickael Latocca (Univ. Evry)

Existence of weak solution to SQG with $L^{4/3}$ initial data

In this talk I will explain how one can construct weak solutions to the SQG with initial data in $L^{4/3}$, which is critical for compactness methods. The solutions we obtain conserve the $H^{-1/2}$ norm : this is a consequence of no anomalous dissipation in the zero viscosity limit. This is a joint work with Luigi De Rosa (GSSI L'Aquila) and Jaemin Park (Yonsei University).

Chenmin Sun (Univ. Créteil)

Observability for the elctromagnetic Schrödinger equation on the two-dimensional torus

In this talk, I will revisit the observability analysis for the Schrödinger equation on the two-dimensional torus, subject to a first-order perturbation by a magnetic potential. It turns out that there is a sufficient and almost necessary geometric control condition for the electromagnetic Schrödinger equation that goes beyond the classical geometric control condition established by Lebeau for the Schrödinger equation. Under this new geometric condition, I will show how a relatively simple analysis leads to a high-frequency observability result on the semiclassical timescale $O(1/h^{3/2})$, which is much shorter than the $O(1/h^2)$ timescale for the purely electronic Schrödinger operator observed from any nonempty open set. This talk is based on joint work with Jingrui Niu (Jussieu) and Kévin Le Balc'h (Jussieu).

Leonardo Tolomeo (Univ. Edinburgh)

Transport of Gaussian measures under the flow of semilinear (S)PDEs : quasi-invariance and singularity.

In this talk, we consider the Cauchy problem for a number of semilinear PDEs, subject to initial data distributed according to a family of Gaussian measures. We first discuss how the flow of Hamiltonian equations transports these Gaussian measures. When the transported measure is absolutely continuous with respect to the initial measure, we say that the initial measure is quasi-invariant. In the high-dispersion regime, we exploit quasi-invariance to build a (unique) global flow for initial data with negative regularity, in a regime that cannot be replicated by the deterministic (pathwise) theory. In the 0-dispersion regime, we discuss the limits of this approach, and exhibit a sharp transition from quasi-invariance to singularity, depending on the regularity of the initial measure. We will also discuss how the same techniques can be used in the context of stochastic PDEs, and how they provide information on the invariant measures for their flow. This is based on joint works with J. Forlano (Monash University) and with J. Coe (University of Edinburgh).

James Wright (Univ. Edinburgh)

A pointwise ergodic theorem

Hui Zhu (New York Univ.)

Schrödinger equation on the torus : controllability and observability in the rough setting

In this talk, I will discuss recent joint work with Nicolas Burq on the controllability and observability of the linear Schrödinger equation on the torus. We establish observability results for a wide class of measurable observation domains and rough potentials. Our approach uses mathematical induction on the dimension and relies on a cluster structure of the space-time frequencies. This structure, originally introduced by Granville and Spencer and later applied by Bourgain in his study of the Schrödinger equation on the torus, plays an important role in our analysis.

Claude Zuily (Univ. Paris-Saclay)

Equipartition of energy for the water-wave system

Equipatition of energy (between kinetic and potential) holds for many pdes. However it has been proved a long time ago by Lord Rayleigh that this does not hold for the water wave system. In this talk we shall show that modifying the kinetic energy we still have equipartition.